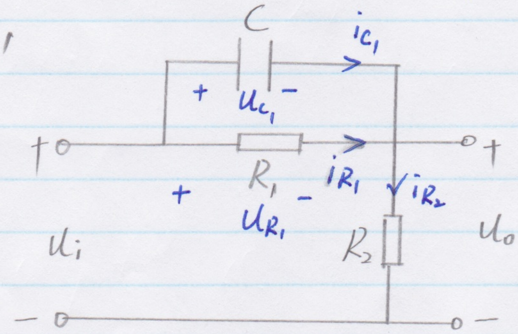
Assignment #1

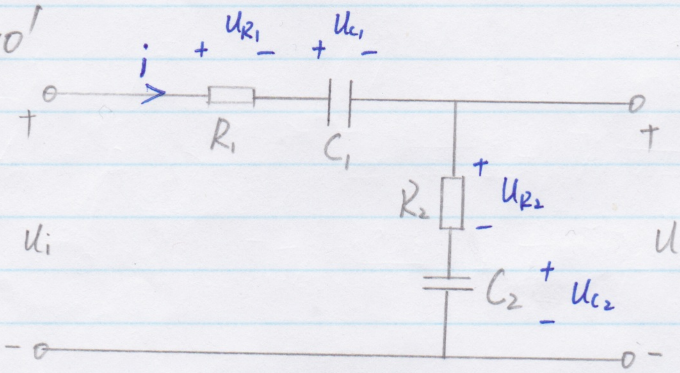
Construct ODEs for the following systems:

1) 10'



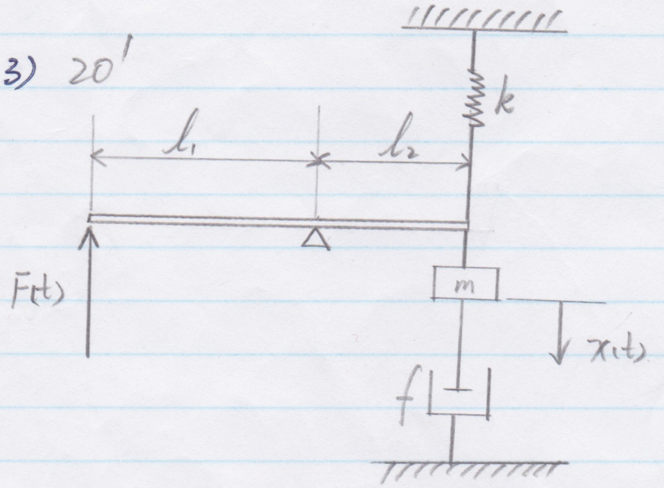
Input: u_i
Output: u_o

2) 10'



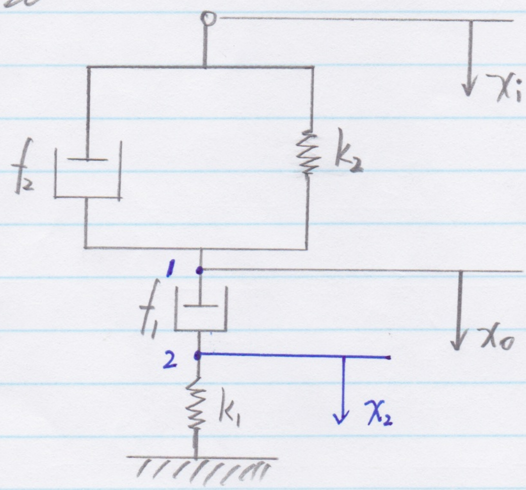
Input: u_i
Output: u_o

3) 20'



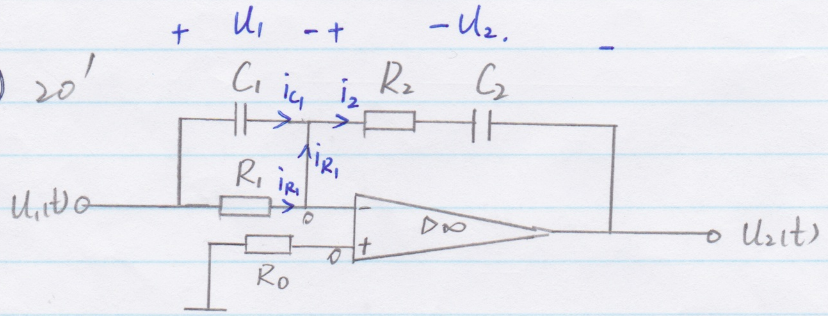
Input: $F(t)$
Output: $x(t)$

4) 20'



Input: x_i
Output: x_0

5) 20'



Input: $U_1(t)$
Output: $U_2(t)$

Solution for Assignment #1.

1) From KCL:

$$i_{C_1} + i_{R_1} = i_{R_2}. \qquad ①$$

The vol tage across R_2 is u_o.

From KVL:

$$u_i = u_o + u_{C_1}$$
$$= u_o + u_{R_1}$$

$$i_{R_1} = \frac{u_{R_1}}{R_1} = \frac{u_i - u_o}{R_1}$$

$$i_{C_1} = C\frac{du_{C_1}}{dt} = C\frac{d(u_i - u_o)}{dt}$$

$$i_{R_2} = \frac{u_o}{R_2}.$$

By using ①. we have.

$$C\frac{d(u_i - u_o)}{dt} + \frac{u_i - u_o}{R_1} = \frac{u_o}{R_2}.$$

$$C\dot{u}_i - C\dot{u}_o + \frac{1}{R_1}u_i - \frac{1}{R_1}u_o = \frac{1}{R_2}u_o$$

$$R_1R_2C\dot{u}_o + (R_1+R_2)u_o = R_1R_2C\dot{u}_i + R_2u_i$$

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2) The current in this circuit is i

For R_2, C_2

$$\left.\begin{aligned} u_{R_2} &= i R_2 \\ u_{R_2} + u_{C_2} &= u_o \end{aligned}\right\} \Rightarrow \quad u_{C_2} = u_o - u_{R_2}$$

For capacitor C_2, we have.

$$i = C_2 \frac{du_{C_2}}{dt} = C_2 \frac{d(u_o - u_{R_2})}{dt}$$

$$= C_2 \frac{d(u_o - iR_2)}{dt}$$

$$= C_2 \dot{u}_o - C_2 R_2 \dot{i}$$

$$\Rightarrow \quad i = C_2 \dot{u}_o - C_2 R_2 \dot{i} \qquad ①$$

For. R_1. C_1

$$\left\{\begin{aligned} u_{R_1} &= i R_1 \\ u_{R_1} + u_{C_1} + u_o &= u_i \end{aligned}\right. \Rightarrow \quad u_{C_1} = u_i - u_o - u_{R_1}$$

For C_1

$$i = C_1 \frac{du_{C_1}}{dt} = C_1 \frac{d(u_i - u_o - u_{R_1})}{dt}$$

$$= C_1 \frac{d(u_i - u_o - iR_1)}{dt}$$

$$= C_1 \dot{u}_i - C_1 \dot{u}_o - C_1 R_1 \dot{i}$$

$$\Rightarrow \quad i = C_1 \dot{u}_i - C_1 \dot{u}_o - C_1 R_1 \dot{i} \qquad ②.$$

From ① & ②., we can have

$$C_2 \dot{u}_o - C_2 R_2 \dot{i} = C_1 \dot{u}_i - C_1 \dot{u}_o - C_1 R_1 \dot{i} \qquad ③ \qquad i = i$$

or. $$\frac{C_2 \dot{u}_o - i}{R_2 C_2} = \frac{C_1 \dot{u}_i - C_1 \dot{u}_o - i}{R_1 C_1} \qquad ④ \qquad \dot{i} = \dot{i}$$

We use ④

$$\Rightarrow R_1C_1C_2\dot{U}_o - R_1C_1 i = R_2C_1C_2\dot{U}_i - R_2C_1C_2\dot{U}_o - R_2C_2 i$$

$$(R_1C_1 - R_2C_2)i = (R_1+R_2)C_1C_2\dot{U}_o - R_2C_1C_2\dot{U}_i$$

$$\Rightarrow i = \frac{(R_1+R_2)C_1C_2\dot{U}_o - R_2C_1C_2\dot{U}_i}{R_1C_1 - R_2C_2} \qquad ⑤$$

Substitute ⑤ into ①

$$\frac{(R_1+R_2)C_1C_2\dot{U}_o - R_2C_1C_2\dot{U}_i}{R_1C_1 - R_2C_2} = C_2\dot{U}_o - C_2R_2\frac{(R_1+R_2)C_1C_2\ddot{U}_o - R_2C_1C_2\ddot{U}_i}{R_1C_1 - R_2C_2}$$

$$(R_1+R_2)C_1C_2\dot{U}_o - R_2C_1C_2\dot{U}_i = (R_1C_1 - R_2C_2)C_2\dot{U}_o - C_2R_2(R_1+R_2)C_1C_2\ddot{U}_o + C_2R_2R_2C_1C_2U$$

$\ddot{U}_i$

$$R_2C_2(C_1+C_2)\dot{U}_o - R_2C_1C_2\dot{U}_i = -C_2R_2(R_1+R_2)C_1C_2\ddot{U}_o + C_2R_2R_2C_1C_2\ddot{U}_i$$

$$(C_1+C_2)\dot{U}_o - C_1\dot{U}_i = -(R_1+R_2)C_1C_2\ddot{U}_o + R_2C_1C_2\ddot{U}_i$$

$$(R_1+R_2)C_1C_2\ddot{U}_o + (C_1+C_2)\dot{U}_o = R_2C_1C_2\ddot{U}_i + C_1\dot{U}_i \qquad ⑥$$

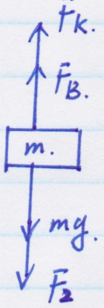
If the initial conditions for U_i & U_o are ° 0.

⑥ becomes

$$(R_1+R_2)C_1C_2\dot{U}_o + (C_1+C_2)U_o = R_2C_1C_2\dot{U}_i + C_1U_i \qquad ⑦.$$

Both ⑥ & ⑦ are correct

3) Free body diagram for "m"



Spring: $F_K = k(x + x_0)$, where $x_0 = mg/k$.

Damper: $F_B = f\,\dot{x}$

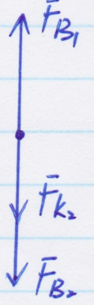
Lever: $F\,l_1 = F_2\,l_2 \Rightarrow F_2 = \frac{l_1}{l_2}F$

Mass: $F_2 + mg - F_K - F_B = m\ddot{x}$

$$\frac{l_1}{l_2}F + mg - kx - kx_0 - f\dot{x} = m\ddot{x}$$

$$m\ddot{x} + f\dot{x} + kx = \frac{l_1}{l_2}F$$

4) First take point 1 to analyze.
suppose the mass of point 1 is m_1 , with $m_1 = 0$



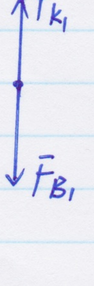
$F_{k_2} = k_2(x_i - x_o)$

$F_{B_2} = f_2(\dot{x}_i - \dot{x}_o)$

$F_{B_1} = f_1(\dot{x}_o - \dot{x}_2)$

$$\left. \begin{array}{l} F_{k_2} + F_{B_2} - F_{B_1} = m_1 \ddot{x}_o \\ \qquad\qquad\qquad\quad = 0. \end{array} \right\} \Rightarrow \begin{array}{l} F_{B_1} = F_{k_2} + F_{B_2} \\ f_1(\dot{x}_o - \dot{x}_2) = k_2(x_i - x_o) + f_2(\dot{x}_i - \dot{x}_o) \quad (1) \end{array}$$

Second, take point 2 to analyze.
suppose the mass of point 2 is m_2 . with $m_2 = 0$



$F_{k_1} = k_1 x_2$

$F_{B_1} = f_1(\dot{x}_o - \dot{x}_2)$

$$\left. \begin{array}{l} F_{B_1} - F_{k_1} = m_2 \ddot{x}_2 \\ \qquad\qquad\quad = 0 \end{array} \right\} \Rightarrow \begin{array}{l} F_{B_1} = F_{k_1} \\ f_1(\dot{x}_o - \dot{x}_2) = k_1 x_2 \quad (2) \end{array}$$

From ① & ②, we have.

$$k_1 x_2 = k_2 (x_i - x_0) + f_2 (\dot{x}_i - \dot{x}_0)$$

$$\Rightarrow x_2 = \frac{k_2}{k_1}(x_i - x_0) + \frac{f_2}{k_1}(\dot{x}_i - \dot{x}_0) \qquad ③$$

Substitute ③ into ②.

$$f_1 \left[\dot{x}_0 - \frac{k_2}{k_1}(\dot{x}_i - \dot{x}_0) - \frac{f_2}{k_1}(\ddot{x}_i - \ddot{x}_0)\right] = k_2 (x_i - x_0) + f_2 (\dot{x}_i - \dot{x}_0)$$

$$f_1 \dot{x}_0 - \frac{f_1 k_2}{k_1}(\dot{x}_i - \dot{x}_0) - \frac{f_1 f_2}{k_1}(\ddot{x}_i - \ddot{x}_0) = k_2 (x_i - x_0) + f_2 (\dot{x}_i - \dot{x}_0)$$

$$k_1 f_1 \dot{x}_0 - k_2 f_1 (\dot{x}_i - \dot{x}_0) - f_1 f_2 (\ddot{x}_i - \ddot{x}_0) = k_1 k_2 (x_i - x_0) + k_1 f_2 (\dot{x}_i - \dot{x}_0)$$

$$\cancel{f_1 f_2 \ddot{x}_0 + (k_2 f_1 + k_1 f_2) \dot{x}_0 + k_1 (f_1 + k_2) x_0}$$

$$f_1 f_2 \ddot{x}_0 + (k_1 f_1 + k_2 f_1 + k_1 f_2) \dot{x}_0 + k_1 k_2 x_0 = f_1 f_2 \ddot{x}_i + (k_1 f_2 + k_2 f_1) \dot{x}_i + k_1 k_2 x_i$$

5) $i_{C_1} = C_1 \dfrac{du_{C_1}}{dt}$

$= C_1 \dfrac{du_1}{dt} = C_1 \dot{u}_1$

$i_{R_1} = \dfrac{u_{R_1}}{R_1} = \dfrac{u_1}{R_1}$

$i_2 = i_{C_1} + i_{R_1} = C_1 \dot{u}_1 + \frac{1}{R_1} u_1$ ①

$u_{R_2} = i_2 R_2.$

$u_{C_2} = -u_2 - i_2 R_2.$

$i_{C_2} = i_2 = C_2 \dfrac{du_{C_2}}{dt}$

$= C_2 \dfrac{d(-u_2 - i_2 R_2)}{dt}$

$= -C_2 \dot{u}_2 - C_2 R_2 \dot{i}_2$ use ①

$C_1 \dot{u}_1 + \frac{1}{R_1} u_1 = -C_2 \dot{u}_2 - C_2 R_2 (C_1 \ddot{u}_1 + \frac{1}{R_1} \dot{u}_1)$

$\cancel{R_2 C_1}$ $C_2 \dot{u}_2 = -C_2 R_2 C_1 \ddot{u}_1 - (C_1 + \frac{C_2 R_2}{R_1}) \dot{u}_1 - \frac{1}{R_1} u_1$

$\cancel{R_1 C_2 \dot{u}_2 = -R_1 R_2 C_1 \ddot{u}_1 - (R_1 C_1 + R_2 C_2) \dot{u}_1 - u_1}$

$R_1 C_2 \dot{u}_2 = -R_1 R_2 C_1 C_2 \ddot{u}_1 - (R_1 C_1 + R_2 C_2) \dot{u}_1 - u_1$

Assignment # 2.

Due: Thursday. 26th. Sept. 2013.
4:00 pm.

1) 10' The mechanical system, as shown in Fig. 1, has $F(t)$ as the input and $x(t)$ as the output.
Find the transfer function $T(s) = X(s)/F(s)$, with zero initial conditions.

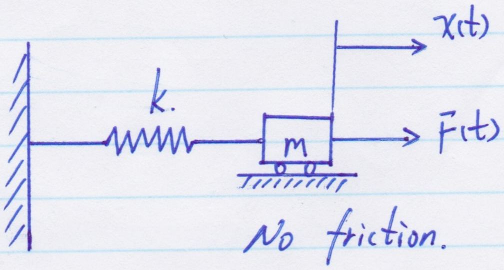


Fig. 1.

2) 10' The mechanical system, as shown in Fig. 2, has $x_i(t)$ as the input and $x_o(t)$ as the output.

Find the transfer function $T(s) = X_o(s)/X_i(s)$, with zero initial conditions. In addition, the gravity can be neglected. but "m" cannot be neglected.

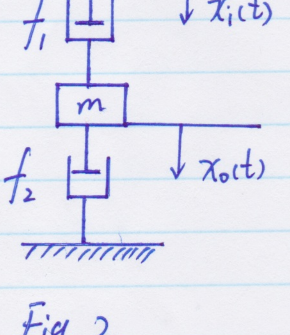


Fig. 2.

20'

3) The height control system of a type of unmanned autonomous vehicle has the following transfer function

$$T(s) = \frac{Y(s)}{R(s)} = \frac{s+1}{(s^2+6s+10)(s+2)}$$

(a) Find the impulse response of the system, that is, $r(t) = \delta(t)$, $y(t) = ?$

(b) Find the unit step response of the system, that is,

$r(t) = u(t)$, $y(t) = ?$

(c) What is the final value of the unit step response?

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Solution for Assignment #2.

1) Newton's 2nd Law:

$$F(t) - k\,x(t) = m\,\ddot{x}(t)$$

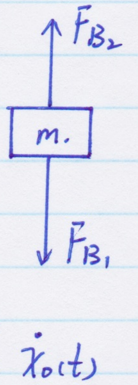
Laplace Transform with zero initial conditions

$$F(s) - kX(s) = ms^2X(s).$$

$$F(s) = (ms^2 + k)\,X(s)$$

$$T(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2+k}.$$

2) Free body diagram of mass "m."



$$F_{B_2}(t) = f_2\,\dot{x}_o(t)$$

$$F_{B_1}(t) = f_1\,(\dot{x}_i(t) - \dot{x}_o(t))$$

$$F_{B_1} \cancel{= F_{B_2} =}$$

$$F_{B_1}(t) - F_{B_2}(t) = m.\,\ddot{x}_o(t)$$

L.T. with zero initial conditions.

$$F_{B_1}(s) - F_{B_2}(s) = ms^2 X_o(s)$$

$$f_1 s X_i(s) - f_1 s X_o(s) - f_2 s X_o(s) = ms^2 X_o(s)$$

$$f_1 s X_i(s) = [ms^2 + (f_1+f_2)s] X_o(s)$$

$$T(s) = \frac{X_o(s)}{X_i(s)} = \frac{f_1 s}{ms^2+(f_1+f_2)s} = \frac{f_1}{ms+f_1+f_2}$$

3)

(a) $r(t) = \delta(t)$, $R(s) = 1$.

$$Y(s) = T(s) \cdot R(s) = T(s)$$

$$= \frac{s+1}{(s^2+6s+10)(s+2)}$$

$$= \frac{a_1 s + a_2}{s^2+6s+10} + \frac{a_3}{s+2}$$

$$a_3 = (s+2)T(s)\Big|_{s=-2} = -\frac{1}{2}$$

$$Y(s) = \frac{a_1 s + a_2}{s^2+6s+10} + \frac{-\frac{1}{2}}{s+2}$$

$$= \frac{a_1 s^2 + 2a_1 s + a_2 s + 2a_2 - \frac{1}{2}s^2 - 3s - 5}{(s^2+6s+10)(s+2)}$$

$$= \frac{s+1}{(s^2+6s+10)(s+2)}$$

$$\begin{cases} -\frac{1}{2}+a_1=0. \\ 2a_1+a_2-3=1 \\ 2a_2-5=1 \end{cases} \Rightarrow \quad \begin{aligned} a_1 &= \tfrac{1}{2} \\ a_2 &= 3. \end{aligned}$$

Thus. $Y(s) = \dfrac{\frac{1}{2}s+3}{s^2+6s+10} - \dfrac{\frac{1}{2}}{s+2}$

$$= \frac{1}{2}\frac{s+6}{(s+3)^2+1} - \frac{1}{2}\frac{1}{s+2}$$

$$= \frac{1}{2}\left[\frac{s+3}{(s+3)^2+1} + \frac{3}{(s+3)^2+1}\right] - \frac{1}{2}\frac{1}{s+2}.$$

$$y(t) = \frac{1}{2}\left[e^{-3t}\cos t + 3e^{-3t}\sin t\right] - \frac{1}{2}e^{-2t} \qquad t \geq 0.$$

or

$$y(t) = \frac{1}{2}\left(e^{-3t}\cos t + 3e^{-3t}\sin t - e^{-2t}\right)u(t).$$

(b) $r(t) = u(t)$, $R(s) = \frac{1}{s}.$

$$Y(s) = R(s)\,T(s) = \frac{s+1}{s(s+2)(s^2+6s+10)}$$

$$= \frac{a_1}{s} + \frac{a_2}{s+2} + \frac{a_3 s + a_4}{s^2+6s+10}$$

$$= \frac{a_1(s+2)(s^2+6s+10) + a_2 s(s^2+6s+10) + (a_3 s + a_4)s(s+2)}{s(s+2)(s^2+6s+10)}$$

$$= \frac{a_1(s^3+6s^2+10s+2s^2+12s+20)+a_2(s^3+6s^2+10s) + a_3 s^3 + 2a_3 s^2 + a_4 s^2 + 2a_4 s.}{s(s+2)(s^2+6s+10)}$$

$$= \frac{(a_1+a_2+a_3)s^3 + (8a_1+6a_2+2a_3+a_4)s^2 + (22a_1+10a_2+2a_4)s + 20a_1}{s(s+2)(s^2+6s+10)}$$

$$= \frac{s+1}{s(s+2)(s^2+6s+10)}$$

$$\begin{cases} a_1+a_2+a_3=0. \\ 8a_1+6a_2+2a_3+a_4=0. \\ 22a_1+10a_2+2a_4=1 \\ 20a_1=1 \end{cases} \Longrightarrow \begin{cases} a_1=\frac{1}{20} \\ a_2=\frac{1}{4} \\ a_3=-\frac{3}{10} \\ a_4=-\frac{13}{10} \end{cases}$$

$$Y(s) = \frac{1}{20}\frac{1}{s} + \frac{1}{4}\frac{1}{s+2} - \frac{1}{10}\frac{3s+13}{(s+3)^2+1}$$

$$= \frac{1}{20}\frac{1}{s} + \frac{1}{4}\frac{1}{s+2} - \frac{1}{10}\left(\frac{3(s+3)}{(s+3)^2+1} + \frac{4}{(s+3)^2+1}\right)$$

$$y(t) = \frac{1}{20} + \frac{1}{4}e^{-2t} - \frac{3}{10}e^{-3t}\cos t - \frac{2}{5}e^{-3t}\sin t \qquad t \geq 0.$$

(c) $y(\infty) = \lim_{t\to\infty} y(t) = \frac{1}{20}$.

Using FVT. ① check poles

$s_1 = 0.$
$s_2 = -2.$
$s_{3,4} = -3 \pm j$

all in Left hand side of complex plane.

③ Apply FVT.

$$f(\infty) = \lim_{s\to 0} s \cdot Y(s) = \lim_{s\to 0} s \cdot \frac{s+1}{s(s+2)(s^2+6s+10)}$$

$$= \frac{1}{20}$$

MECH 380 Automatic Control Engineering: Assignment III

Due: 4:00 pm, Oct. 7th, 2013

I. Problem 1 $10'$

The system block diagram is as shown in Fig. 1. Find the transfer function from $R(s)$ to $Y(s)$. You

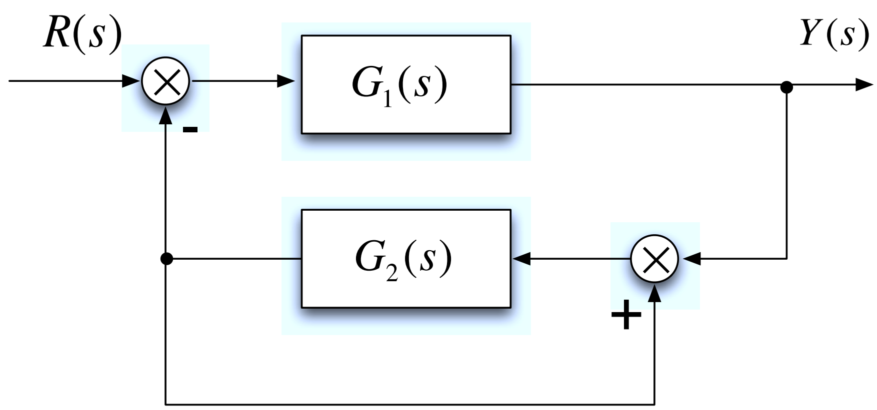


Fig. 1. *System block diagram.*

are required to obtain

$$\Phi(s) = \frac{Y(s)}{R(s)}.$$

II. Problem 2 $30'$

The system block diagram is as shown in Fig. 2. Find the transfer function from $R(s)$ to $Y(s)$ and from $F(s)$ to $Y(s)$, respectively. You are required to obtain

$$\Phi_1(s) = \frac{Y(s)}{R(s)},$$

and

$$\Phi_2(s) = \frac{Y(s)}{F(s)}.$$

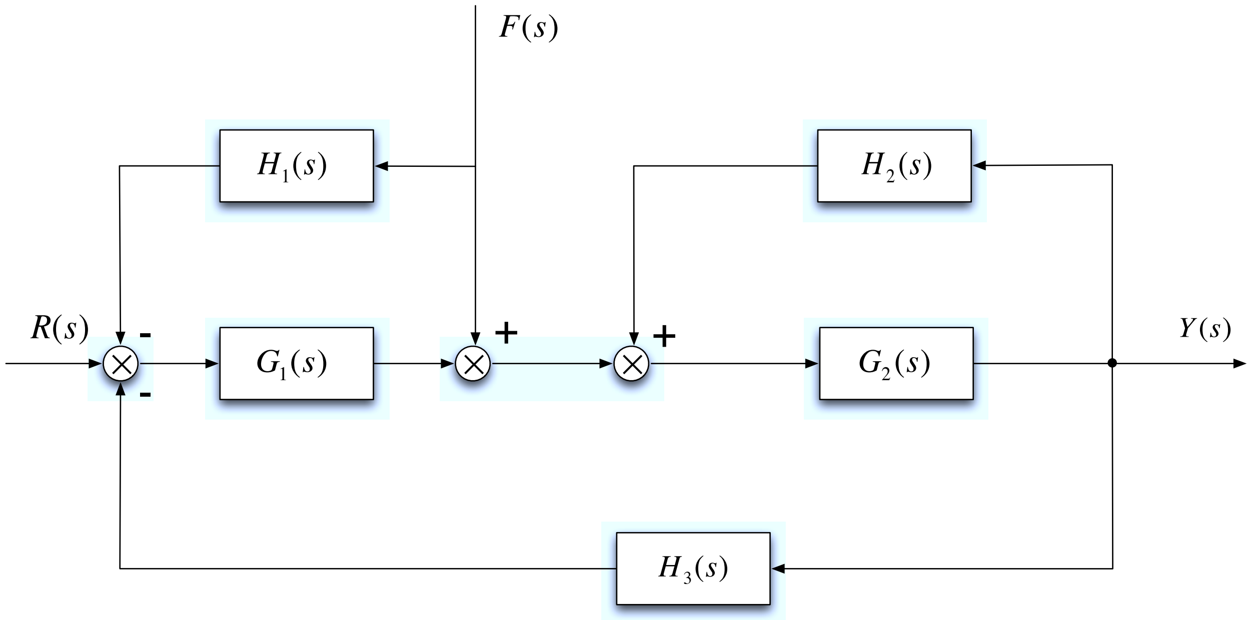


Fig. 2. *System block diagram.*

III. Problem 3 $20'$

The cloased-loop transfer function of a first-order system can be represented by

$$\Phi(s) = \frac{K}{Ts+1}, \tag{1}$$

where K and T are positive constants. The impulse response of this system can be shown in Fig. 3. Try to determine K and T.

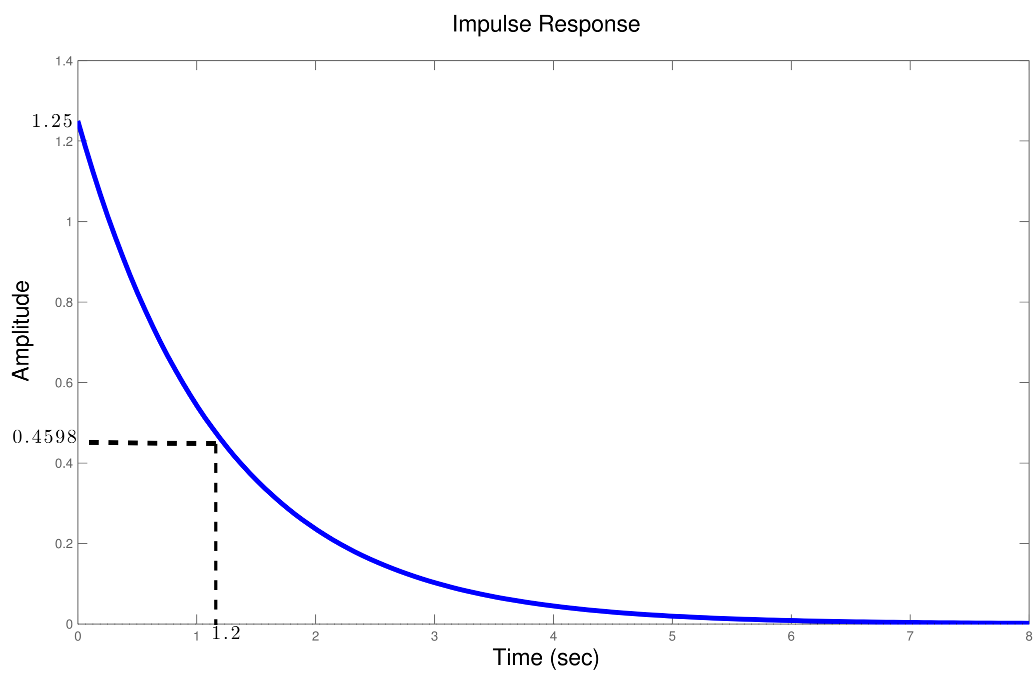
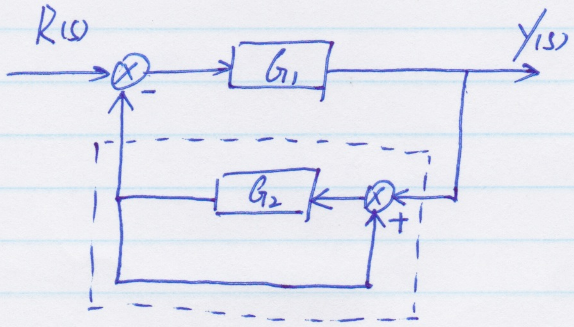


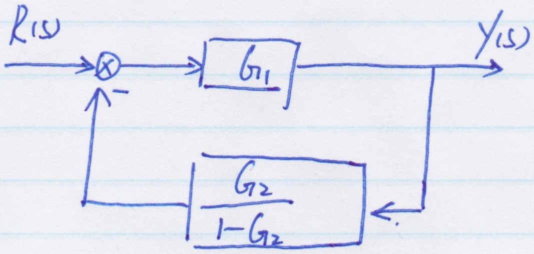
Fig. 3. *Impulse response of the 1st-order system.*

Solution for Assignment III

Problem 1:



$\Rightarrow$

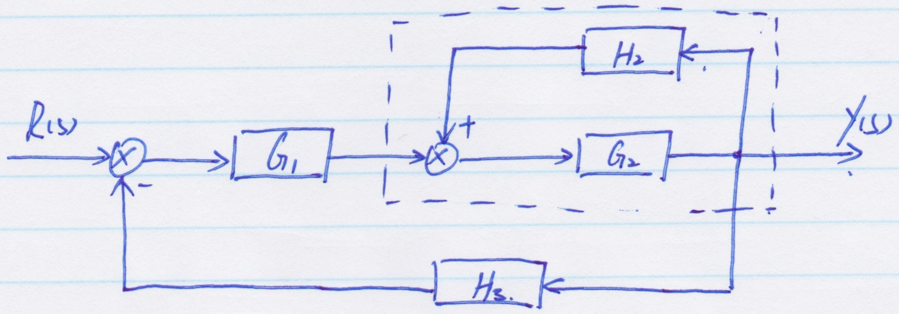


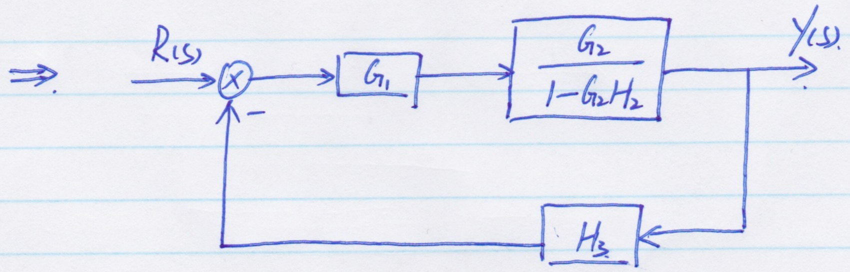
$$\Phi(s) = \frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1 \frac{G_2}{1-G_2}}$$

$$= \frac{G_1(1-G_2)}{1-G_2+G_1G_2}$$

Problem 2:

① $\Phi_1(s) = \frac{Y(s)}{R(s)}$. in this case $F(s)=0$. Thus, the original block diagram becomes.

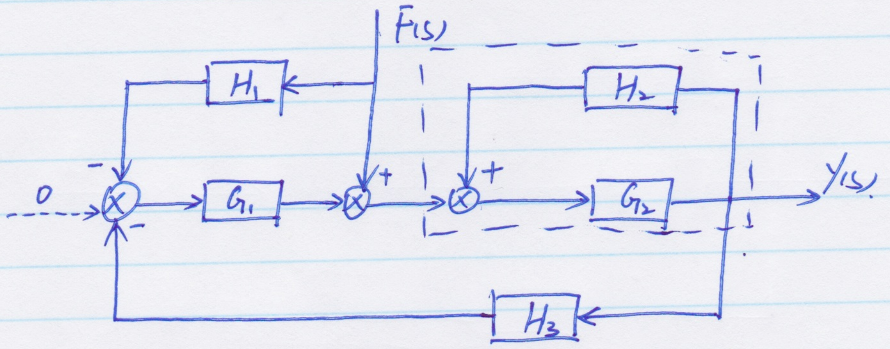




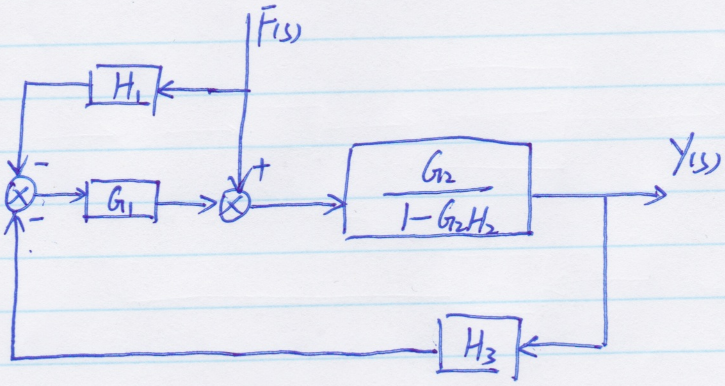
$$\Phi_1(s) = \frac{Y(s)}{R(s)} = \frac{\frac{G_1 G_2}{1-G_2H_2}}{1+\frac{G_1G_2}{1-G_2H_2}\cdot H_3}$$

$$= \frac{G_1 G_2}{1-G_2H_2+G_1G_2H_3}$$

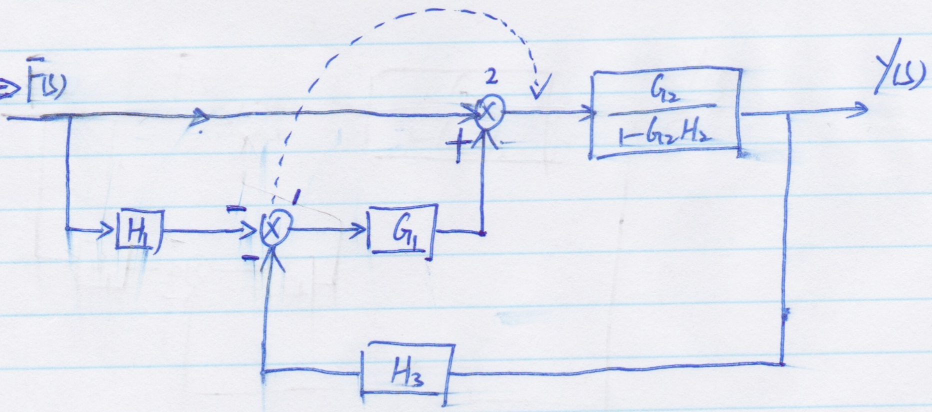
② $\Phi_2(s) = \dfrac{Y(s)}{F(s)}$, in this case. $R(s)=0$. Thus the original block diagram becomes

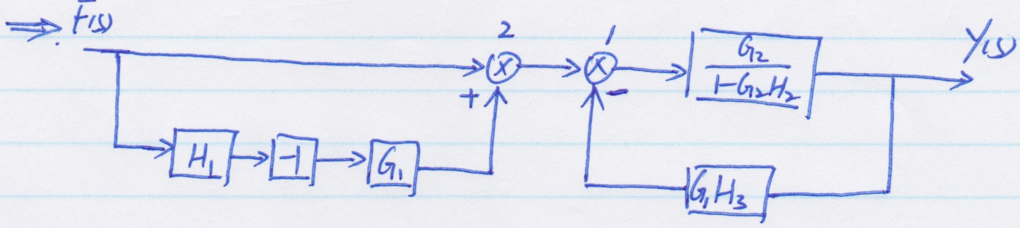


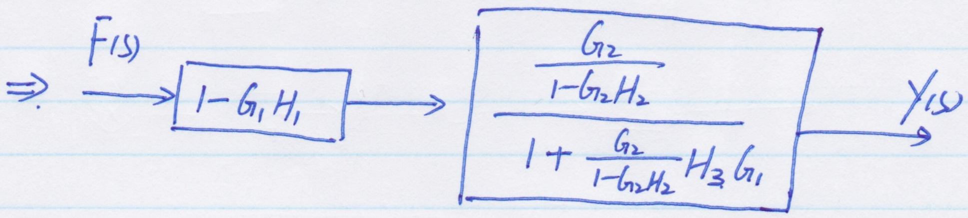
⇒



⇒







$$\Phi_2(s) = \frac{Y(s)}{F(s)} = (1-G_1H_1)\,\frac{G_2}{1-G_2H_2+G_1G_2H_3}$$

$$= \frac{G_2(1-G_1H_1)}{1-G_2H_2+G_1G_2H_3}.$$

Problem 3:

The transfer function $\Phi(s) = \frac{Y(s)}{R(s)} = \frac{K}{TS+1}$.

With the impulse signal, which means $r(t) = \delta(t)$, $R(s)=1$

Then. $Y(s) = \Phi(s) R(s) = \Phi(s) = \frac{K}{TS+1}$.

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{K}{TS+1}\right\}.$$

$$= \mathcal{L}^{-1}\left\{\frac{K/T}{S+1/T}\right\}.$$

$$= \frac{K}{T} e^{-t/T}, \quad t \geq 0.$$

① When $t=0$. $y(t) = y(0) = \frac{K}{T}$, as shown in Fig. 3.

$$\frac{K}{T} = 1.25 \qquad (1)$$

② when $t=T$ $y(t) = y(T) = \frac{K}{T} e^{-1} = 0.368 \frac{K}{T}$

$$= 0.368 \times 1.25$$

$$\approx 0.4598.$$

Thus, if $y(t) = 0.4598$, time t is the time constant T. From the figure, we have. $T = 1.2$.

From equation (1), $K = 1.25 \times T = 1.5$.

Solution for Assignment IV.

Problem 1:

Transfer function for inner loop

$$T_1(s) = \frac{\frac{10}{s(s+1)}}{1+\frac{10}{s(s+1)}\tau s}$$

$$= \frac{10}{s^2+s+10\tau s}$$

$$= \frac{10}{s(s+1+10\tau)}$$

Thus the open-loop transfer function for this system is

$$G(s) = K \cdot T_1(s)$$

$$= \frac{10K}{s(s+1+10\tau)}$$

General form of a 2nd-order system's open-loop transfer function is

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

Thus $\begin{cases} 10K = \omega_n^2 \\ 1+10\tau = 2\zeta\omega_n \end{cases} \Rightarrow \begin{cases} \omega_n = \sqrt{10K} \\ \zeta = \dfrac{1+10\tau}{2\sqrt{10K}} \end{cases} \Rightarrow \begin{cases} K = \dfrac{\omega_n^2}{10} \\ \tau = \dfrac{2\zeta\omega_n - 1}{10} \end{cases}$

$$\sigma_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} = 0.163$$

$$\frac{-\zeta}{\sqrt{1-\zeta^2}}\pi = \ln 0.163 = -1.81$$

$$\frac{\zeta^2}{1-\zeta^2} = \left(\frac{1.81}{\pi}\right)^2 = 0.33$$

$$\zeta^2 = 0.33 - 0.33\zeta^2$$

$$(1+0.33)\zeta^2 = 0.33$$

$$\zeta = 0.498$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{1-\zeta^2}\,\omega_n} = 1$$

$$\omega_n = \frac{\pi}{\sqrt{1-\zeta^2}} = 3.62$$

$$K = \frac{\omega_n^2}{10} = 1.31$$

$$\tau = \frac{2\zeta\omega_n - 1}{10} = 0.26$$

Problem 2:

$$G(s) = \frac{\cancel{0.16}\ 1}{s(s+0.6)}$$

General form.

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

$$\begin{cases} \omega_n^2 = 1 \\ 2\zeta\omega_n = 0.6 \end{cases} \Rightarrow \begin{matrix} \omega_n = 1 \\ \zeta = 0.3 \end{matrix}$$

$$\sigma_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} = e^{-\frac{0.3}{\sqrt{1-0.09}}\pi} = 0.3723 = 37.23\%$$

$$t_r = \frac{\pi-\theta}{\omega_n\sqrt{1-\zeta^2}} = 1.97 \qquad \theta = \arctan\frac{\sqrt{1-\zeta^2}}{\zeta} = 1.266$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{1-\zeta^2}\,\omega_n} = \frac{\pi}{\sqrt{1-0.09}} = 3.29$$

$$t_s = \begin{cases} \frac{4}{\zeta\omega_n} = 13.3 & \Delta = 0.02 \\ \frac{3}{\zeta\omega_n} = 10 & \Delta = 0.05 \end{cases}$$

Problem 3:

From the figure.

$$\sigma_p = \frac{1.1-1}{1} \times 100\% = 10\%.$$

$$t_p = 0.1$$

$$\sigma_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} = 0.1$$

$$\frac{-\zeta}{\sqrt{1-\zeta^2}}\pi = \ln 0.1 = -2.303.$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}}\pi = 2.303.$$

$$\frac{\zeta^2}{1-\zeta^2} = 0.538.$$

$$\zeta^2 = 0.538 - 0.538\zeta^2$$

$$\zeta^2 = 0.35$$

$$\zeta = 0.59.$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{1-\zeta^2}\omega_n} = 0.1$$

$$\omega_n = \frac{\pi}{\sqrt{1-\zeta^2}\ 0.1} = 38.9.$$

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} = \frac{1513.2}{s(s+45.9)}.$$

Solution for Assignment #5

1. Solution:

Time domain: $y(t) = 1 + e^{-t} - e^{-2t} \quad (t \geq 0)$

Frequency domain: $Y(s) = \frac{1}{s} + \frac{1}{s+1} - \frac{1}{s+2}$.

$$Y(s) = \frac{1}{s} + \frac{1}{s+1} - \frac{1}{s+2}$$

$$= \frac{(s+1)(s+2) + s(s+2) - s(s+1)}{s(s+1)(s+2)}$$

$$= \frac{s^2+3s+2+s^2+2s-s^2-s}{s(s+1)(s+2)}$$

$$= \frac{s^2+4s+2}{s(s+1)(s+2)}$$

The transfer function $\Phi(s) = \frac{Y(s)}{R(s)} = \frac{Y(s)}{1/s} = \frac{s^2+4s+2}{(s+1)(s+2)}$

Two poles at -1, -2. both are negative.

$\Rightarrow$ The system is stable

2. Solution: $D(s) = s^4 + 2s^3 + s^2 + 2s + 1 = 0$.

C_1 ✓

C_2 ✓

Routh Table:

s^4	1	1	1
s^3	2	2	
s^2	0	1	
s^1			
s^0			

First element in s^2 row is 0.

① Try to use $s = \frac{1}{x}$

$$D(x) = \frac{1}{x^4} + \frac{2}{x^3} + \frac{1}{x^2} + \frac{2}{x} + 1 = 0$$

$$= x^4 + 2x^3 + x^2 + 2x + 1 = 0.$$

$D(x)$ is exactly the same as $D(s)$, it does not help

② Try to multiply $s+a$ to $D(s)$.

$\bar{D}(s) = D(s)(s+a)$ choose $a = 2$.

$$= D(s)(s+2)$$
$$= (s^4 + 2s^3 + s^2 + 2s + 1)(s+2)$$
$$= s^5 + 2s^4 + 2s^4 + 4s^3 + s^3 + 2s^2 + 2s^2 + 4s + s + 2.$$

$$= s^5 + 4s^4 + 5s^3 + 4s^2 + 5s + 2$$

Routh Table:

s^5	1	5	5
s^4	4	4	2
s^3	4	4.5	
s^2	-0.5	2	
s^1	20.5		
s^0	2.		

The system is not stable. two positive (real) / complex with positive real parts poles exist.

3. Solution: Open-loop Transfer Function

$$G(s) = \frac{K}{s(0.1s+1)(0.25s+1)}$$

Characteristic equation:

$$D(s) = s(0.1s+1)(0.25s+1) + K. = 0.$$

$$\Leftrightarrow \quad D(s) = s(s+10)(s+4) + 40K = 0.$$

$$= s(s^2+14s+40) + 40K$$

$$= s^3+14s^2+40s+40K$$

Define $s = z-2$. As long as $Re(z) < 0$, $Re(s)$ will be smaller than -2, i.e., s locates on the left hand side of $-2 \pm j\omega$.

$$D(z) = (z-2)^3 + 14(z-2)^2 + 40(z-2) + 40K.$$

$$= z^3-6z^2+12z-8+14z^2-56z+56+40z-80+40K$$

$$= z^3+8z^2-4z-32+40K$$

C_1 ✓

C_2 Fail. $\Rightarrow$ There is no such a K that can satisfy the requirement

Solution for Assignment #6.

1. $R(s) = \frac{5}{s^2+25}$

$\Phi_e(s) = \frac{1}{1+G(s)}$

$= \frac{1}{1+\frac{100}{s(0.1s+1)}}$

$= \frac{s(0.1s+1)}{s(0.1s+1)+100}$

Characteristic equation $D(s) = s(0.1s+1) + 100 = 0.$

C_1 ✓

C_2 ✓

Routh Table

s^2	0.1	100
s^1	1	
s^0	100	

This system is stable. steady state error exists.

$e_{ss} = \lim_{s\to 0} s \cdot R(s)\,\Phi_e(s)$

$= \lim_{s\to 0} s \cdot \frac{5}{s^2+25} \frac{s(0.1s+1)}{s(0.1s+1)+100}$

$= 0.$

2. 1) The closed-loop transfer function from $R(s)$ to $Y(s)$

$$\Phi(s) = \frac{\frac{20}{0.05s+1}\frac{1}{s+5}}{1+\frac{20}{0.05s+1}\frac{1}{s+5}2.5}$$

$$= \frac{20.}{(0.05s+1)(s+5)+50}$$

characteristic equation $D(s) = (0.05s+1)(s+5)+50$

$$= 0.05s^2 + 1.25s + 50 = 0.$$

C_1 ✓

C_2 ✓

Routh Table

s^2	0.05	50
s^1	1.25	
s^0	50	

This system is stable, steady state error exists.

Transfer function from $F(s)$ to $E(s)$.

$$\Phi_{fe}(s) = \frac{-\frac{1}{s+5}2.5}{1+\frac{20}{0.05s+1}\frac{1}{s+5}2.5}$$

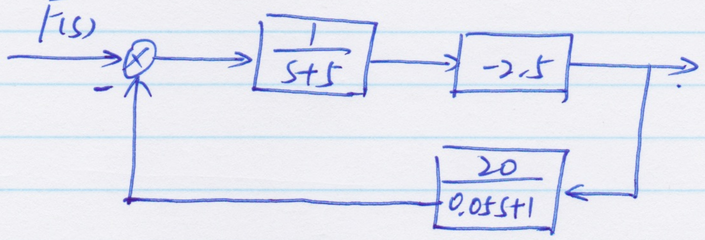
$$= \frac{-2.5(0.05s+1)}{(0.05s+1)(s+5)+50.}$$

$\cancel{e_{ssf} = \lim_{s\to 0} s\, F(s).}$

$$e_{ssf} = \lim_{s\to 0} s \cdot F(s) \cdot \Phi_{fe}(s) = \lim_{s\to 0} s \cdot \frac{2}{s} \cdot \frac{-2.5(0.05s+1)}{(0.05s+1)(s+5)+50}$$

$$= 2 \cdot \frac{-2.5}{5+50} = \frac{-5}{55} = -\frac{1}{11}$$

Block diagram



2) If add $\frac{1}{s}$ before $F(s)$, we need to check the stability again.

$$\Phi(s) = \frac{\frac{1}{s}\frac{20}{0.05s+1}\frac{1}{s+5}}{1+\frac{1}{s}\frac{20}{0.05s+1}\frac{1}{s+5}2.5}$$

$$= \frac{20}{s(s+5)(0.05s+1)+50.}$$

$$D(s) = s(s+5)(0.05s+1)+50$$

$$= s(0.05s^2+1.25s+1)+50$$

$$= 0.05s^3+1.25s^2+s+50=0.$$

C_1 ✓

C_2 ✓

Routh Table:

s^3	0.05	1
s^2	1.25	50.
s^1	1	
s^0	50	

This system is stable. steady state error exists.

$$\Phi_{fe}(s) = \frac{-\frac{1}{s+5}\,2{,}5}{1+\frac{20}{0{,}05s+1}\cdot\frac{1}{s+5}\cdot\frac{1}{s}\,2{,}5}$$

$$= \frac{-2{,}5s(0{,}05s+1)}{s(s+5)(0{,}05s+1)+50.}$$

$$e_{ssf} = \lim_{s\to 0.} s\cdot F(s)\cdot \Phi_{fe}(s)$$

$$= \lim_{s\to 0} s\cdot\frac{2}{s}\cdot\frac{-2{,}5s(0{,}05s+1)}{s(s+5)(0{,}05s+1)+50}$$

$$=0.$$

3). The characteristic equation is the same as in 2).
Thus, steady state error exists.

$$\Phi_{fe}(s) = \frac{-\frac{1}{s}\frac{1}{s+5}\,2{,}5}{1+\frac{20}{0{,}05s+1}\cdot\frac{1}{s+5}\cdot\frac{1}{s}\,2{,}5}$$

$$= \frac{-2{,}5(0{,}05s+1)}{s(s+5)(0{,}05s+1)+50.}$$

$$e_{ssf} = \lim_{s\to 0.} s\cdot F(s)\cdot \Phi_{fe}(s)$$

$$= \lim_{s\to 0} s\cdot\frac{2}{s}\cdot\frac{-2{,}5(0{,}05s+1)}{s(s+5)(0{,}05s+1)+50}$$

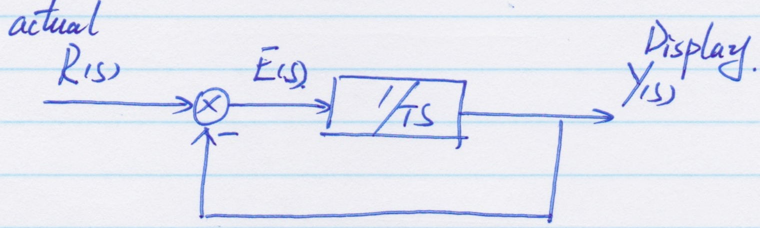
$$= \frac{-5}{50} = -\frac{1}{10}$$

4) By adding $\frac{1}{s}$ before $F(s)$, the steady state error under step input can be eliminated; Adding $\frac{1}{s}$ after $F(s)$ cannot.

3. Closed-loop transfer function $1/(TS+1)$.

$\Rightarrow$ Open-loop transfer function $G(s) = \frac{1}{TS}$

$$\Phi(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{1}{TS}}{1+\frac{1}{TS}} = \frac{1}{TS+1}$$



R(s) is a step signal $R(s) = \frac{A}{S}$
$r(t) = A u(t)$

For this 1st-order system $y(t) = A(1-e^{-t/T}),\ t \geq 0$.

After 1 min = 60 seconds. $y(60) = 98\% A$

$= A(1-e^{-60/T})$

$\Rightarrow \quad 1-e^{-60/T} = 0.98$

$\Rightarrow \quad T = 15.35$

If the increasing rate is $10°C/min = \frac{1}{6}\ °C/s$.

$\Rightarrow \quad r(t) = \frac{1}{6}t$
$R(s) = \frac{1}{6}\frac{1}{S^2}$

This system is Type I system. $K_v = K = \frac{1}{T}$

$$e_{ss} = \frac{1}{K_v}\frac{1}{6} = \frac{1}{6}T = 2.55°C.$$

2. 1) The closed-loop transfer function from $R(s)$ to $Y(s)$

$$\Phi(s) = \frac{\frac{20}{0.05s+1}\frac{1}{s+5}}{1+\frac{20}{0.05s+1}\frac{1}{s+5}2.5}$$

$$= \frac{20.}{(0.05s+1)(s+5)+50}$$

characteristic equation $D(s) = (0.05s+1)(s+5)+50$

$$= 0.05s^2 + 1.25s + 50 = 0.$$

C_1 ✓

C_2 ✓

Routh Table

s^2	0.05	$\cancel{25}$ 50
s^1	1.25	
s^0	$\cancel{25}$ 50	

This system is stable, steady state error exists.

Transfer function from $F(s)$ to $E(s)$.

$$\Phi_{fe}(s) = \frac{-\frac{1}{s+5}2.5}{1+\frac{20}{0.05s+1}\frac{1}{s+5}2.5}$$

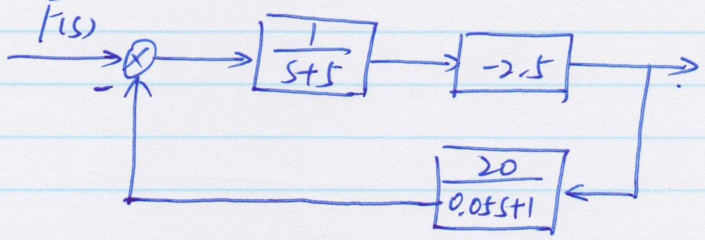
$$= \frac{-2.5(0.05s+1)}{(0.05s+1)(s+5)+50.}$$

$\cancel{e_{ssf} = \lim_{s\to 0} s\, F(s).}$

$$e_{ssf} = \lim_{s\to 0} s \cdot F(s) \cdot \Phi_{fe}(s) = \lim_{s\to 0} s \cdot \frac{2}{s} \cdot \frac{-2.5(0.05s+1)}{(0.05s+1)(s+5)+50}$$

$$= 2 \cdot \frac{-2.5}{5+50} = \frac{-5}{55} = -\frac{1}{11}$$

Block diagram



2) If add $\frac{1}{s}$ before F(s), we need to check the stability again.

$$\Phi(s) = \frac{\frac{1}{s}\frac{20}{0.05s+1}\frac{1}{s+5}}{1+\frac{1}{s}\frac{20}{0.05s+1}\frac{1}{s+5}2.5}$$

$$= \frac{20}{s(s+5)(0.05s+1)+50.}$$

$$D(s) = s(s+5)(0.05s+1)+50$$

$$= s(0.05s^2+1.25s+1)+50$$

$$= 0.05s^3+1.25s^2+s+50=0.$$

C_1 ✓

C_2 ✓

Routh Table:

s^3	0.05	1
s^2	1.25	50.
s^1	1	
s^0	50	

This system is stable. steady state error exists.

$$\Phi_{fe}(s) = \frac{-\frac{1}{s+5} 2.5}{1+\frac{20}{0.05s+1}\frac{1}{s+5}\frac{1}{s}2.5}$$

$$= \frac{-2.5s(0.05s+1)}{s(s+5)(0.05s+1)+50}.$$

$$e_{ssf} = \lim_{s\to 0} s \cdot F(s) \cdot \Phi_{fe}(s)$$

$$= \lim_{s\to 0} s \cdot \frac{2}{s} \cdot \frac{-2.5s(0.05s+1)}{s(s+5)(0.05s+1)+50}$$

$$= 0.$$

3). The characteristic equation is the same as in 2).
Thus, steady state error exists.

$$\Phi_{fe}(s) = \frac{-\frac{1}{s}\frac{1}{s+5} 2.5}{1+\frac{20}{0.05s+1}\frac{1}{s+5}\frac{1}{s}2.5}$$

$$= \frac{-2.5(0.05s+1)}{s(s+5)(0.05s+1)+50}.$$

$$e_{ssf} = \lim_{s\to 0} s \cdot F(s) \cdot \Phi_{fe}(s)$$

$$= \lim_{s\to 0} s \cdot \frac{2}{s} \cdot \frac{-2.5(0.05s+1)}{s(s+5)(0.05s+1)+50}$$

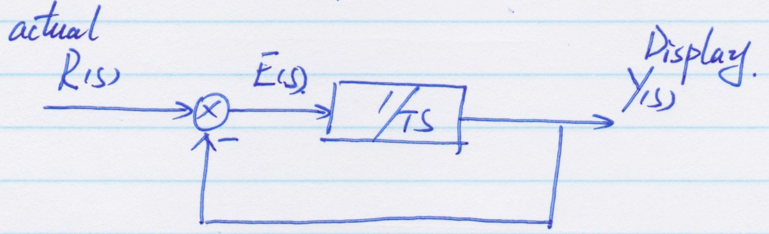
$$= \frac{-5}{50} = -\frac{1}{10}$$

4) By adding $\frac{1}{s}$ before $F(s)$, the steady state error under step input can be eliminated; Adding $\frac{1}{s}$ after $F(s)$ cannot.

3. Closed-loop transfer function $1/TS+1$.

$\Rightarrow$ Open-loop transfer function $G(s) = \frac{1}{TS}$

$$\Phi(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{1}{TS}}{1+\frac{1}{TS}} = \frac{1}{TS+1}$$



~~Ass~~ R(s) is a step signal $R(s) = \frac{A}{s}$

$r(t) = A u(t)$

For this 1st-order system $y(t) = A(1 - e^{-t/T}),\ t \geq 0.$

After 1 min = 60 seconds. $y(60) = 98\% A$

$= A(1 - e^{-60/T})$

$\Rightarrow \quad 1 - e^{-60/T} = 0.98$

$\Rightarrow \quad T = 15.35$

~~If R(t)~~

If the increasing rate is $10°C/min = \frac{1}{6}°C/s$.

$\Rightarrow \quad r(t) = \frac{1}{6}t$

$R(s) = \frac{1}{6}\frac{1}{s^2}$

This system is Type I system. $K_v = K = \frac{1}{T}$

$$e_{ss} = \frac{1}{K_v}\frac{1}{6} = \frac{1}{6}T = 2.55°C.$$

Solution for Assignment #7.

1. ① $Y(s) = \frac{1}{s} - 1.8\frac{1}{s+4} + 0.8\frac{1}{s+9}$

$= \frac{36}{s(s+4)(s+9)}$

Thus, the transfer function is

$\Phi(s) = \frac{36}{(s+4)(s+9)}$ → This system is stable

$x(t) = \sin\omega t$

$\Phi(j\omega) = \frac{36}{(j\omega+4)(j\omega+9)}$

$= \frac{36}{36-\omega^2+13\omega j}$

$|\Phi(j\omega)| = \frac{36}{\sqrt{(36-\omega^2)^2+(13\omega)^2}}$

$\omega = 4.$

$|\Phi(j\omega)| = \cancel{0.6923}\ 0.6462$

$\varphi = \angle\Phi(j\omega) = -\arctan\frac{13\omega}{36-\omega^2} = \cancel{-1.1760}\ -1.2036$

$= \cancel{-67.38^\circ}\ -68.96^\circ$

Thus, $y_{ss}(t) = |\Phi(j\omega)|\sin(\omega t+\varphi)$

$= \cancel{0.6923}\sin(4t - \cancel{67.38^\circ})$
$0.6462 \quad 68.96^\circ$

② $\omega = 7.$

$|\Phi(j\omega)| = \cancel{0.4235}\ 0.39$

$\varphi = \angle\Phi(j\omega) = -\arctan\frac{13\omega}{36-\omega^2} - \pi = \cancel{-1.7243}\ -1.7127$

$= \cancel{-98.79^\circ}\ -98.13^\circ$

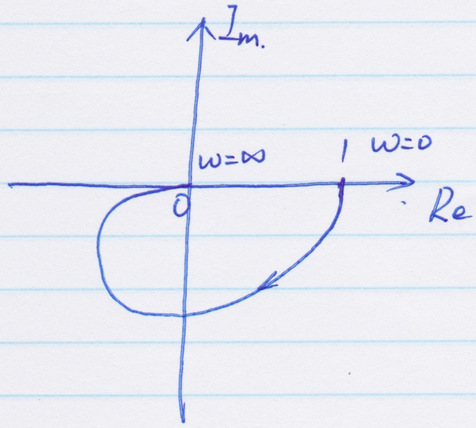
Thus, $y_{ss}(t) = \cancel{0.4235}\sin(7t - \cancel{98.79^\circ})$
$0.39 \quad 98.13^\circ$

③ $\bar{\Phi}(s) = \dfrac{36}{(s+4)(s+9)} = \dfrac{1}{(\frac{1}{4}s+1)(\frac{1}{9}s+1)}$ $\qquad$ $\bar{\Phi}(j\omega) = \dfrac{1}{(\frac{1}{4}j\omega+1)(\frac{1}{9}j\omega+1)}$

There is no integral, when $\omega=0$ $\quad |\bar{\Phi}(j\omega)| = 1$ $\quad \angle\bar{\Phi}(j\omega)=0$

$\omega=\infty$ $\quad |\bar{\Phi}(j\omega)| = 0$ $\quad \angle\bar{\Phi}(j\omega) = -180°$

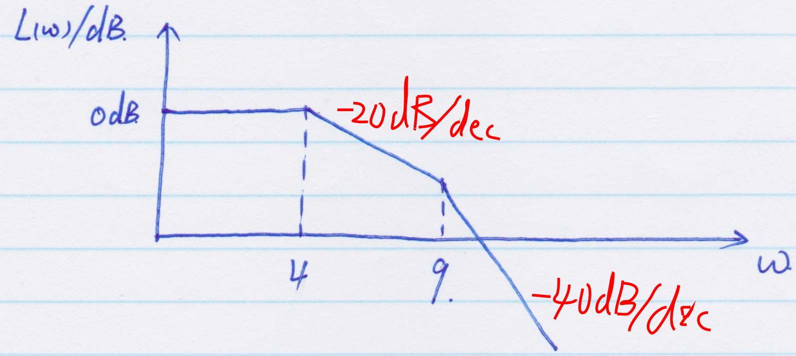
Nyquist Plot



④ Corner frequency $\quad$ 4 rad/s $\quad$ 9 rad/s.

-20dB/dec $\quad$ -20dB/dec.

There is no gain or integral, so in the low frequency area, $L(\omega) = 0$ dB.



⑤ At corner frequency, in the approximated Bode Plot, $L(4) = 0$ dB.

From ①, $20\lg|\bar{\Phi}(j\omega)| = 20\lg \cancel{0.6923}\ 0.6462 = -3.\cancel{1944}79$ dB.

Thus, the error between the actual & approximated ones is $-3.\cancel{1944}79$ dB

2. ~~Gain~~ ~~K=20~~

$$T(s) = \frac{\frac{35}{3}(s+\frac{1}{7})}{s(s+\frac{1}{3})(s^2+\frac{1}{2}s+\frac{1}{4})}$$

$$= \frac{20(7s+1)}{s(3s+1)(4s^2+2s+1)}$$

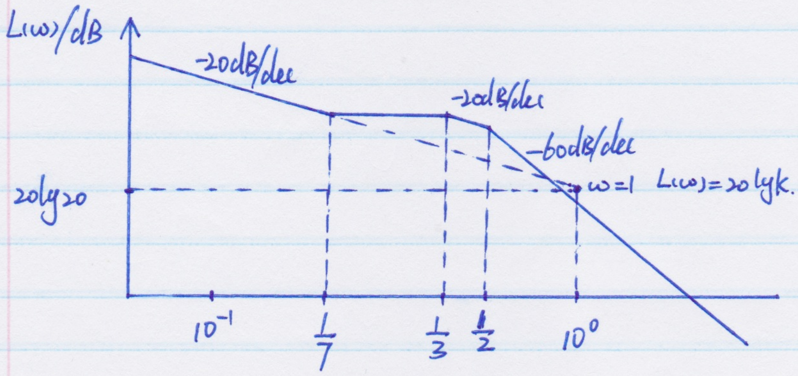
Gain $K=20.$

Integral $v=1$

1st-order derivative $\omega_1 = \frac{1}{7}$ +20dB/dec

Inertia $\omega_2 = \frac{1}{3}$ −20dB/dec

Oscillation $\omega_3 = \frac{1}{2}$ −40dB/dec.



3. From the Bode Plot, there is no $\frac{1}{s}$ but a Gain.

$20\lg K = 40$ $K=100.$

Two inertia exist.

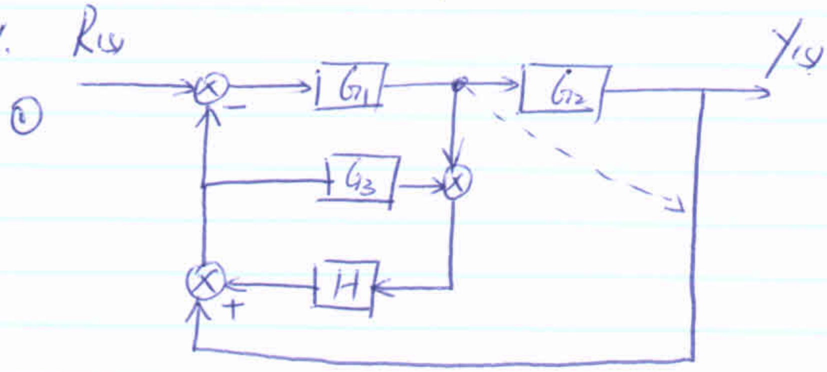
$$T(s) = \frac{K}{(\frac{1}{\omega_1}s+1)(\frac{1}{\omega_2}s+1)} = \frac{100\omega_1\omega_2}{(s+\omega_1)(s+\omega_2)}$$

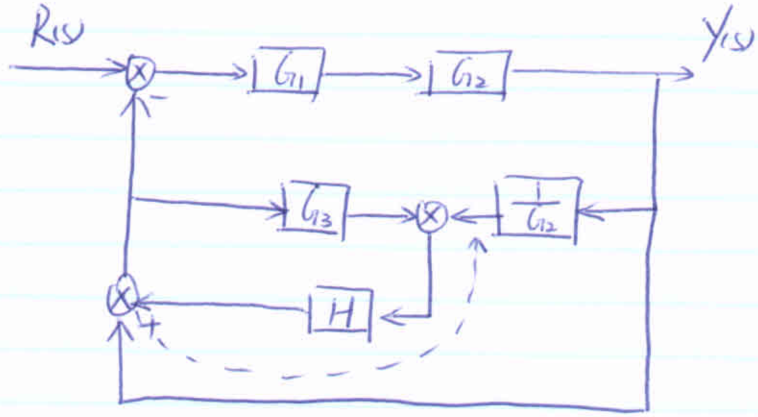
4. From the Bode Plot, except for the given gain K, there exist two integrals, one 1st-order derivative and one inertia.

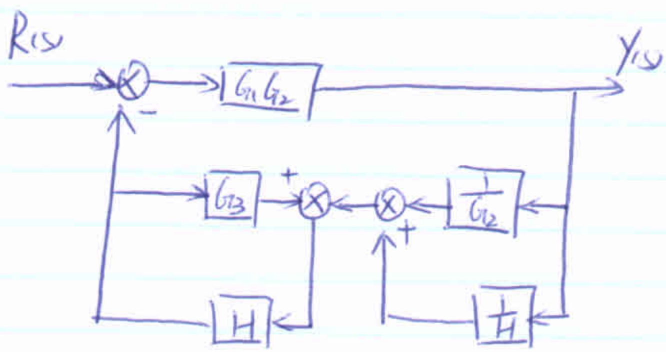
$$T(s) = \frac{K(\frac{1}{w_1}s+1)}{s^2(\frac{1}{w_2}s+1)}$$

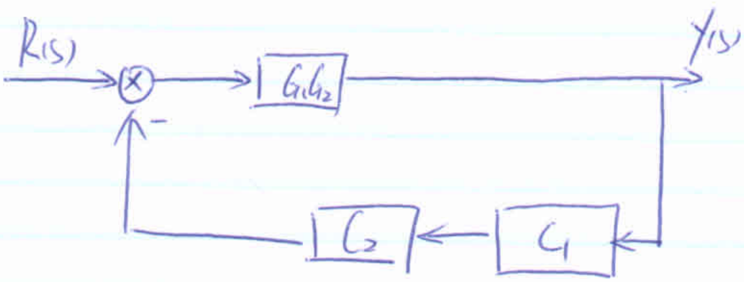
Solution for Assignment #8

1.









$$C_1 = \frac{1}{G_2} + \frac{1}{H}$$

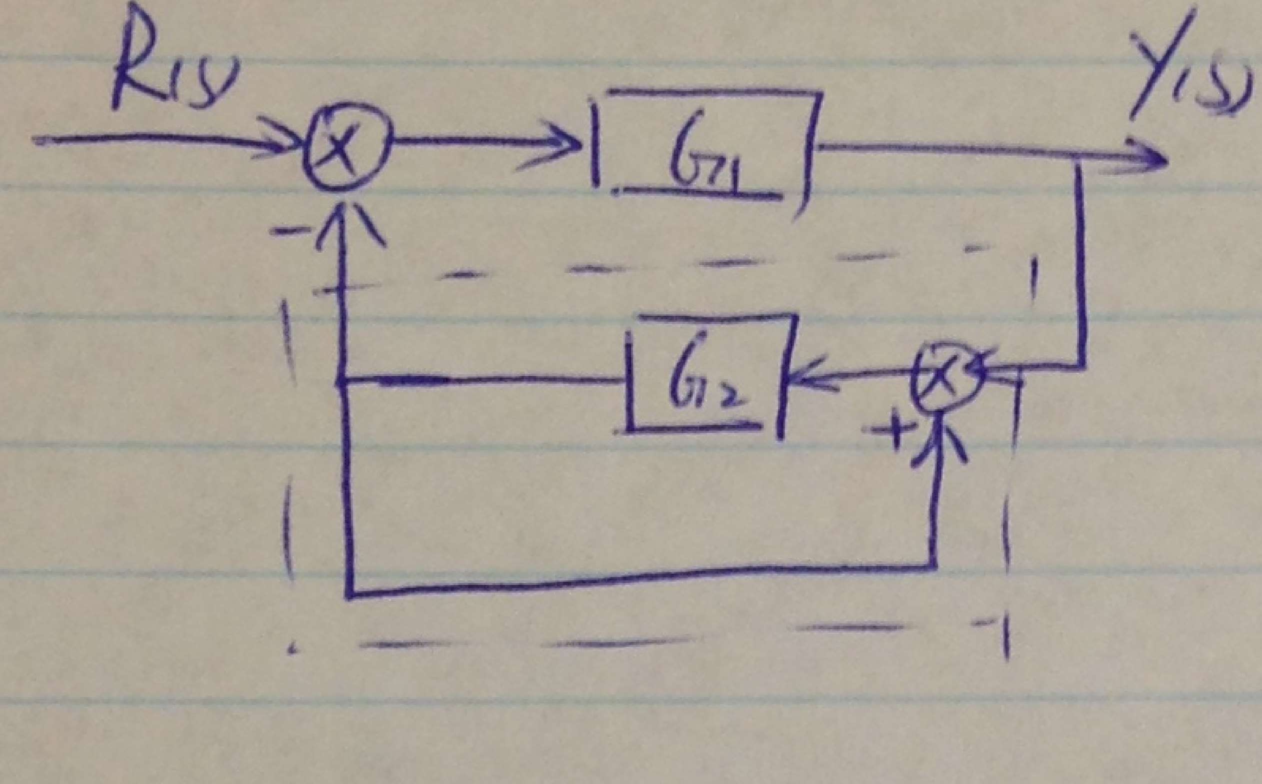
$$\cancel{C_1 = \frac{\frac{1}{G_2}}{1 - \frac{1}{G_2}H} = \frac{H}{G_2 H - 1}}$$

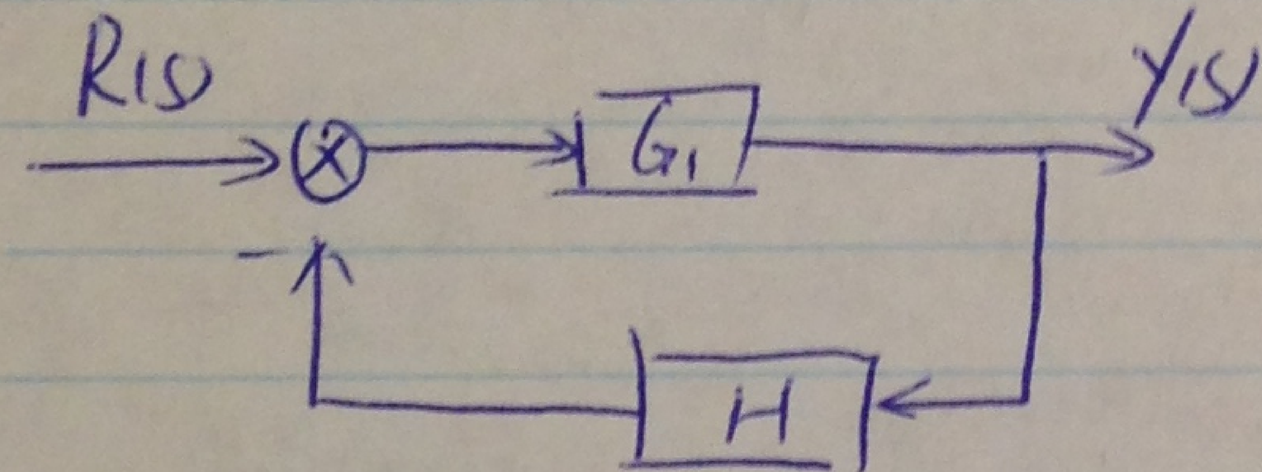
$$C_2 = \frac{H}{1 - G_3 H}$$

$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2}{1+G_1G_2C_1C_2}=\cancel{\frac{G_1G_2}{1+G_1G_2\frac{H^2}{(G_2H-1)(1-G_3H)}}}=\frac{G_1G_2(1-G_3H)}{1-G_3H+G_1H+G_1G_2}$$

$$=\cancel{\frac{G_1G_2(G_2H-1)(1-G_3H)}{(G_2H-1)(1-G_3H)+G_1G_2H^2}}$$

②





$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1}{1+G_1H}$$

$$H=\frac{G_2}{1-G_2}\qquad T(s)=\frac{G_1}{1+G_1\frac{G_2}{1-G_2}}$$

$$=\frac{G_1(1-G_2)}{1-G_2+G_1G_2}$$

2. The transfer function from $R(s)$ to $E(s)$ is

$$\Phi_e(s)=\frac{1-\frac{\lambda_2s^2+\lambda_1s}{Ts+1}\frac{K_2}{s(s+2\xi)}}{1+\frac{K_1K_2}{s(s+2\xi)}}$$

$$=\frac{Ts^3+(2\xi T+1)s^2+2\xi s-K_2\lambda_2s^2-K_2\lambda_1s}{s(s+2\xi)(Ts+1)+K_1K_2(Ts+1)}$$

stability check.

$$D(s) = Ts^3 + (2\zeta T+1)s^2 + 2\zeta s + K_1K_2Ts + K_1K_2$$
$$= 0.2s^3 + 1.2s^2 + 21s + 100.$$

C_1 ✓

C_2 ✓

s^3	0.2	21
s^2	1.2	100
s^1	$\frac{1.2\times 21-20}{1.2}$	0
s^0	$100.$	

C_3 ✓.

System is stable.

To make it a type III system

$$\begin{cases} 2\zeta T+1 - K_2\lambda_2 = 0. \\ 2\zeta - K_2\lambda_1 = 0 \end{cases} \Rightarrow \begin{cases} \lambda_2 = \dfrac{2\zeta T+1}{K_2} = 0.024 \\ \lambda_1 = \dfrac{2\zeta}{K_2} = 0.02 \end{cases}$$

3. Routh Table

s^6	1	-4	-7	10
s^5	1 (4)	1 (4)	-2 (-8)	
s^4	-5 (-1)	-5 (-1)	10 (2)	
s^3	0 (-4) (-2)	0 (-2) (-1)	0	
s^2	$-\frac{1}{2}$	2		
s^1	-9			
s^0	2			

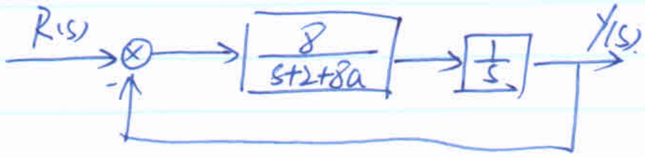
$$F(s) = -s^4 - s^2 + 2.$$
$$F'(s) = -4s^3 - 2s$$

Two positive poles.

$F(s)=0.$ $\qquad s^4+s^2-2=0.$

$$s_{1,2} = \pm\sqrt{2}\,j$$
$$s_{3,4} = \pm 1.$$

4.



$$\Phi(s) = \frac{8}{s^2+(2+8a)s+8}$$

① $a=0.$

$$\Phi(s) = \frac{8}{s^2+2s+8}$$

$$\omega_n = \sqrt{8} \text{ rad/s.}$$
$$\zeta = \frac{1}{\omega_n} = \frac{1}{\sqrt{8}}$$

② $\omega_n = \sqrt{8}$

$2+8a = 2\zeta\omega_n. \qquad \zeta = 0.7$

$$a = \frac{2\zeta\omega_n - 2}{8}$$
$$= \frac{0.7\sqrt{8}-1}{4}$$

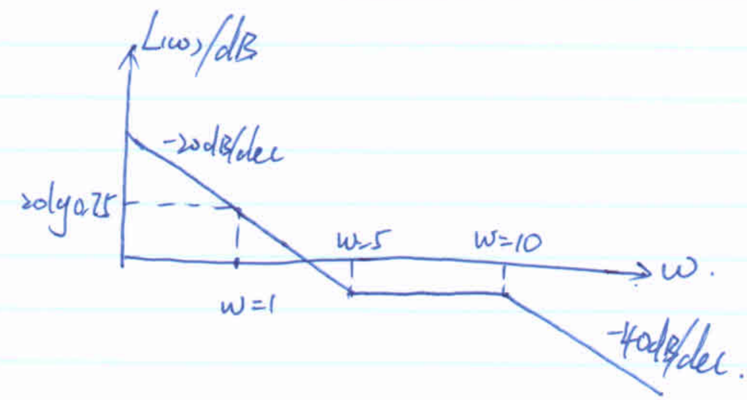
5.

$$G(s)H(s) = \frac{75(0.2s+1)}{100s(\frac{1}{10}^2 s^2 + \frac{16}{100}s+1)}$$

$$= \frac{0.75(0.2s+1)}{s(\frac{1}{10}^2 s^2 + \frac{16}{100}s+1)}$$

Integral $\frac{1}{s}$ Gain 0.75

1st-order	$T=0.2$	$\omega=5$	$+90°$	$+20$ dB/dec
Oscillation	$T=0.1$	$\omega=10$	$-180°$	-40 dB/dec



6. $$G(s) = \frac{10(10s+1)(0.1s+1)}{s^3(0.01s+1)}$$